

Winslow Township School District
Mathematics Curriculum – Algebra I
Unit 3

Overview	Standards for Mathematical Content		Unit Focus	Standards for Mathematical Practice
Unit 3 Quadratic Equations, Functions & Polynomials	<ul style="list-style-type: none"> ● A.APR.A.1 ● A.SSE.A.2 ● A.REI.B.4 ● A.CED.A.1 ● F.IF.B.4* ● F.IF.B.5* ● A.SSE.B.3 ● F.BF.A.1 	<ul style="list-style-type: none"> ● F.IF.C.7* ● F.IF.C.8* ● F.IF.C.9* ● F.IF.B.6 ● F.LE.A.3 ● F.BF.B.3 ● A.REI.D.11 ● A.APR.B.3 ● N.RN.B.3 	<ul style="list-style-type: none"> ● Perform arithmetic operations on polynomials ● Understand the relationship between zeros and factors ● Interpret the structure of expressions ● Solve equations and inequalities in one variable ● Create equations that describe numbers or relationships ● Interpret functions that arise in applications in terms of the context ● Represent and solve equations and inequalities graphically ● Build a function that models a relationship between two quantities ● Construct & compare linear, quadratic, & exponential models ● Build new functions from existing functions ● Analyze functions using different representations ● Use properties of rational and irrational numbers 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments and critique the reasoning of others.</p> <p>MP.4 Model with mathematics.</p>
Unit 3: Suggested Open Educational Resources	A.APR.A.1 Powers of 11 A.SSE.A.2 Equivalent Expressions A.REI.B.4 Visualizing Completing the Square A.REI.B.4 Braking Distance A.REI.B.4 Two Squares are Equal F.IF.B.4 Words – Tables - Graphs F.IF.B.5 The restaurant	A.SSE.B.3 Profit of a company A.SSE.B.3 Rewriting a Quadratic Expression F.IF.C.7a Graphs of Quadratic Functions F.IF.C.8a Springboard Dive F.IF.C.8a Which Function? F.IF.B.9 Throwing Baseballs F.IF.B.6 Mathemafish Population	F.LE.A.3 Population and Food Supply F.BF.B.3 Identifying Even and Odd Functions F.BF.B.3 Transforming the graph of a function A.REI.D.11 Introduction to Polynomials – College Fund A.APR.B.3 Graphing from Factors 1 N.RN.B.3 Operations with Rational and Irrational Numbers	<p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>

Major Supporting Additional (Identified by PARCC Model Content Frameworks).

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Curriculum Unit 3	Standards		Pacing	
			Days	Unit Days
Unit 3 Quadratic Equations, Functions & Polynomials	<ul style="list-style-type: none"> ● A.APR.A.1 ● A.SSE.A.2 ● A.SSE.B.3 	<p>Add, subtract, and multiply polynomials, relating these to arithmetic operations with integers. Factor to produce equivalent forms of quadratic expressions in one variable.</p> <p>Use factoring and completing the square to produce equivalent forms of quadratic expressions in one variable that highlight particular properties such as the zeros or the maximum or minimum value of the function.</p>	13	45
	<ul style="list-style-type: none"> ● A.SSE.A.2 ● A.REI.B.4 ● A.SSE.B.3 ● A.APR.B.3 ● F.IF.C.7* ● F.IF.C.8* ● F.IF.B.6 ● F.IF.B.4* 	<p>Use the structure of an expression to identify ways to rewrite it.</p> <p>Solve quadratic equations in one variable.</p> <p>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p> <p>Graph quadratic functions by hand in simple cases and with technology in complex cases, showing intercepts, extreme values and symmetry of the graph.</p> <p>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.</p> <p>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>	13	
	<ul style="list-style-type: none"> ● F.IF.B.4* ● F.IF.B.5* ● F.IF.B.6 ● F.IF.C.7* ● F.IF.C.9* ● F.BF.A.1 ● F.LE.A.3 ● F.BF.B.3 	<p>Interpret key features of quadratic functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a quadratic function, showing key features and relating the domain of the function to its graph.</p> <p>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p> <p>Graph quadratic functions by hand in simple cases and with technology in complex cases, showing intercepts, extreme values and symmetry of the graph. Compare properties of two quadratic functions, each represented in a different way.</p> <p>Write a function that describes a relationship between two quantities.</p> <p>Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function</p> <p>Identify the effects of transformations and combinations of transformations [$f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$] on a function; find the value of k given the graph.</p>	9	
	<ul style="list-style-type: none"> ● A.REI.D.11 	<p>Find approximate solutions of $f(x) = g(x)$, where $f(x)$ is a linear function and $g(x)$ is a quadratic function by making a table of values, using technology to graph and finding successive approximations.</p>	3	
	<ul style="list-style-type: none"> ● N.RN.B.3 	<p>Explain and justify conclusions about sums and products of rational and irrational numbers.</p>	3	
Assessment, Re-teach and Extension			4	

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Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
<ul style="list-style-type: none"> ● A.APR.A.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. ● A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i> 	<p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Polynomials form a system analogous to the integers. ● Polynomials are closed under the operations of addition, subtraction, and multiplication. <p>Students are able to:</p> <ul style="list-style-type: none"> ● add and subtract polynomials. ● multiply polynomials. ● recognize numerical expressions as a difference of squares and rewrite the expression as the product of sums/differences. ● recognize polynomial expressions in one variable as a difference of squares and rewrite the expression as the product of sums/differences. <p>Learning Goal 1: Add, subtract, and multiply polynomials, relating these to arithmetic operations with integers. Factor to produce equivalent forms of quadratic expressions in one variable.</p>
<ul style="list-style-type: none"> ● A.REI.B.4. Solve quadratic equations in one variable. A.REI.B.4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. A.REI.B.4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.3 Construct viable arguments and critique the reasoning of others.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Multiple methods for solving quadratic equations. ● Transforming a quadratic equation into the form $(x - p)^2 = q$ yields an equation having the same solutions. <p>Students are able to:</p> <ul style="list-style-type: none"> ● use the method of completing the square to transform a quadratic equation in x into an equation of the form $(x - p)^2 = q$. ● derive the quadratic formula from $(x - p)^2 = q$. ● solve a quadratic equations in one variable by inspection. ● solve quadratic equations in one variable by taking square roots. ● solve a quadratic equations in one variable by completing the square. ● solve a quadratic equations in one variable using the quadratic formula. ● solve a quadratic equations in one variable by factoring. ● strategically select, as appropriate to the initial form of the equation, a method for solving a quadratic equation in one variable. ● write complex solutions of the quadratic formula in $a \pm bi$ form.

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<p>write them as $a \pm bi$ for real numbers a and b.</p>		<ul style="list-style-type: none"> analyze the quadratic formula, recognizing the conditions leading to complex solutions (discriminant). <p>Learning Goal 2: Derive the quadratic formula by completing the square and recognize when there are no real solutions.</p> <p>Learning Goal 3: Solve quadratic equations in one variable using a variety of methods (including inspection, taking square roots, factoring, completing the square, and the quadratic formula) and write complex solutions in $a \pm bi$ form.</p>
<ul style="list-style-type: none"> A.CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions. 	<p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p>	<p>Concept(s): No new concept(s) introduced</p> <p>Students are able to:</p> <ul style="list-style-type: none"> create quadratic equations in one variable. use quadratic equations to solve real world problems. <p>Learning Goal 4: Create quadratic equations in one variable and use them to solve problems.</p>
<ul style="list-style-type: none"> F.IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers</i> 	<p>MP.4 Model with mathematics.</p> <p>MP.6 Attend to precision.</p>	<p>Concept(s): No new concept(s) introduced</p> <p>Students are able to:</p> <ul style="list-style-type: none"> interpret maximum/minimum and intercepts of quadratic functions from graphs and tables in the context of the problem. sketch graphs of quadratic functions given a verbal description of the relationship between the quantities. identify intercepts and intervals where function is increasing/decreasing determine the practical domain of a function. <p>Learning Goal 5: Interpret key features of quadratic functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a quadratic function, showing key features and relating the domain of the function to its graph.</p>

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<p><i>would be an appropriate domain for the function</i></p>		
<ul style="list-style-type: none"> ● A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A.SSE.B.3a. Factor a quadratic expression to reveal the zeros of the function it defines. A.SSE.B.3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.4 Model with mathematics.</p> <p>MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Alternate, equivalent forms of a quadratic expression may reveal specific attributes of the function that it defines. <p>Students are able to:</p> <ul style="list-style-type: none"> ● factor a quadratic expression for the purpose of revealing the zeros of a function. ● complete the square for the purpose of revealing the maximum or minimum of a function. <p>Learning Goal 6: Use factoring and completing the square to produce equivalent forms of quadratic expressions in one variable that highlight particular properties such as the zeros or the maximum or minimum value of the function.</p>
<ul style="list-style-type: none"> ● F.BF.A.1. Write a function that describes a relationship between two quantities. F.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context. 	<p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.4 Model with mathematics.</p>	<p>Concept(s): No new concept(s) introduced</p> <p>Students are able to:</p> <ul style="list-style-type: none"> ● given a context, write explicit expressions, a recursive process or steps for calculation for quadratic relationships. <p>Learning Goal 7: Given a context, write an explicit expression, a recursive process or steps for calculation for quadratic relationships.</p>
<ul style="list-style-type: none"> ● F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. F.IF.C.7a. Graph linear and quadratic functions and show intercepts, maxima, and minima. *<i>[emphasize quadratic functions]</i> ● F.IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.3 Construct viable arguments and critique the reasoning of others.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s): No new concept(s) introduced</p> <p>Students are able to:</p> <ul style="list-style-type: none"> ● graph quadratic functions expressed symbolically. ● graph more complicated cases of quadratic functions using technology. ● identify and describe key features of the graphs of quadratic functions. ● given two quadratic functions, each represented in a different way, compare the properties of the functions. <p>Learning Goal 8: Graph quadratic functions by hand in simple cases and with technology in complex cases, showing intercepts, extreme values and symmetry of the graph. Compare properties of two quadratic functions, each represented in a different way.</p>

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<p>F.IF.C.8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <ul style="list-style-type: none"> ● F.IF.C.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i> 		
<ul style="list-style-type: none"> ● F.IF.B.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ● F.LE.A.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. 	<p>MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● A quantity increasing exponentially eventually exceeds a quantity increasing quadratically. <p>Students are able to:</p> <ul style="list-style-type: none"> ● calculate the rate of change of a quadratic function from a table of values or from a function presented symbolically. ● estimate the rate of change from a graph of a quadratic function. ● analyze graphs and tables to compare rates of change of exponential and quadratic functions. <p>Learning Goal 9: Calculate and interpret the average rate of change of a quadratic function presented symbolically or as a table. Estimate and compare the rates of change from graphs of quadratic and exponential functions.</p>
<ul style="list-style-type: none"> ● F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and 	<p>MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Characteristics of even and odd functions in graphs and algebraic expressions ● Vertical and horizontal shifts <p>Students are able to:</p>

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<p>illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p>	<p>MP.7 Look for and make use of structure.</p>	<ul style="list-style-type: none"> ● perform transformations on graphs of linear and quadratic functions. ● identify the effect on the graph of replacing $f(x)$ by <ul style="list-style-type: none"> - $f(x) + k$; - $k f(x)$; - $f(kx)$; - and $f(x + k)$ for specific values of k (both positive and negative). ● identify the effect on the graph of combinations of transformations. ● given the graph, find the value of k. ● illustrate an explanation of the effects on linear and quadratic graphs using technology. ● recognize even and odd functions from their graphs and from algebraic expressions for them. <p>Learning Goal 10: Identify the effects of transformations and combinations of transformations [$f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$] on a function; find the value of k given the graph.</p>
<ul style="list-style-type: none"> ● A.REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.5 Use appropriate tools strategically.</p>	<p>Concept(s): No new concept(s) introduced Students are able to:</p> <ul style="list-style-type: none"> ● approximate the solution(x) to a system of equations comprised of a linear and a quadratic function by using technology to graph the functions, by making a table of values and/or by finding successive approximations. <p>Learning Goal 11: Find approximate solutions of $f(x) = g(x)$, where $f(x)$ is a linear function and $g(x)$ is a quadratic function by making a table of values, using technology to graph and finding successive approximations.</p>
<ul style="list-style-type: none"> ● A.APR.B.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph 	<p>MP.7 Look for and make use of structure.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● General shape(s) and end behavior of cubic functions <p>Students are able to:</p> <ul style="list-style-type: none"> ● find the zeros of a polynomial (quadratic and cubic).

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<p>of the function defined by the polynomial. *[Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available]</p>		<ul style="list-style-type: none"> ● test domain intervals to determine where $f(x)$ is greater than or less than zero. ● use zeros of a function to sketch a graph. <p>Learning Goal 12: Identify zeros of cubic functions when suitable factorizations are available and use the zeros to construct a rough graph of the function. (*cubic functions are presented as the product of a linear and a quadratic factor)</p>
<ul style="list-style-type: none"> ● N.RN.B.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. 	<p>MP.3 Construct viable arguments and critique the reasoning of others.</p> <p>MP.6 Attend to precision.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● The sum or product of two rational numbers is rational. ● The sum of a rational number and an irrational number is irrational. ● The product of a nonzero rational number and an irrational number is irrational. <p>Students are able to:</p> <ul style="list-style-type: none"> ● explain and justify conclusions regarding sums and products of two rational numbers.. ● explain and justify conclusions regarding the sum of a rational and irrational number. ● explain and justify conclusions regarding the product of a nonzero rational and irrational number. <p>Learning Goal 13: Explain and justify conclusions about sums and products of rational and irrational numbers.</p>

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School/District Formative Assessment Plan	School/District Summative Assessment Plan
Pre-Assessment, Quizzes Exit Tickets Daily Monitoring Linkit!	Unit Benchmark Linkit! Diagnostic
Focus Mathematical Concepts	
<p><u>Prerequisite skills:</u></p> <ul style="list-style-type: none"> ● Understanding and use the formal mathematical language of functions. ● Provide students an opportunity to compare two functions (quadratic and exponential), represented in different ways (table, graph, or situation). <p><u>Common Misconceptions:</u></p> <ul style="list-style-type: none"> ● A.SSE.A.2 <p>Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols.</p> <p>Use of real- world context examples to demonstrate the meaning of the parts of algebraic expressions is needed to counter this misconception.</p> <p>Students may also believe that an expression cannot be factored because it does not fit into a form they recognize. They need help with reorganizing the terms until structures become evident.</p> <p>Students will often combine terms that are not like terms. For example, $2 + 3x = 5x$ or $3x + 2y = 5xy$.</p> <p>Students sometimes forget the coefficient of 1 when adding like terms. For example, $x + 2x + 3x = 5x$ rather than $6x$.</p> <p>Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than $5x$.</p> <p>Students will forget to distribute to all terms when multiplying. For example, $6(2x + 1) = 12x + 1$ rather than $12x + 6$.</p> <p>Students may not follow the Order of Operations when simplifying expressions. For example, $4x^2$ when $x = 3$ may be incorrectly evaluated as $4 \cdot 3^2 = 122 = 144$, rather than $4 \cdot 9 = 36$. Another common mistake occurs when the distributive property should be used prior to adding/subtracting. For example, $2 + 3(x - 1)$ incorrectly becomes $5(x - 1) = 5x - 5$ instead of $2 + 3(x - 1) = 2 + 3x - 3 = 3x - 1$.</p> <p>Students fail to use the property of exponents correctly when using the distributive property. For example, $3x(2x - 1) = 6x - 3x = 3x$ instead of simplifying as $3x(2x - 1) = 6x^2 - 3x$.</p>	

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Students fail to understand the structure of expressions. For example, they will write $4x$ when $x = 3$ is 4^3 instead of $4x = 4 \cdot x$ so when $x = 3$, $4x = 4 \cdot 3 = 12$. In addition, students commonly miscalculate $-3^2 = 9$ rather than $-3^2 = -9$.

Students routinely see -3^2 as the same as $(-3)^2 = 9$. A method that may clear up the misconception is to have students rewrite as $-x^2 = -1 \cdot x^2$ so they know to apply the exponent before the multiplication of -1 .

Students frequently attempt to “solve” expressions. Many students add “= 0” to an expression they are asked to simplify. Students need to understand the difference between an equation and an expression.

Students commonly confuse the properties of exponents, specifically the product of powers property with the power of a power property. For example, students will often simplify $(x^2)^3 = x^5$ instead of x^6 .

Students will incorrectly translate expressions that contain a difference of terms. For example, 8 less than 5 times a number is often incorrectly translated as $8 - 5n$ rather than $5n - 8$.

● **A.APR.A.1**

Some students will apply the distributive property inappropriately. Emphasize that it is the *distributive property of multiplication over addition*. For example, the distributive property can be used to rewrite $2(x + y)$ as $2x + 2y$, because in this product the second factor is a sum (i.e., involving addition). But in the product $2(xy)$, the second factor, (xy) , is itself a product, not a sum.

Some students will still struggle with the arithmetic of negative numbers. Consider the expression $(-3) \cdot (2 + (-2))$. On the one hand, $(-3) \cdot (2 + (-2)) = (-3) \cdot (0) = 0$. But using the distributive property, $(-3) \cdot (2 + (-2)) =$

$(-3) \cdot (2) + (-3) \cdot (-2)$. Because the first calculation gave 0, the two terms on the right in the second calculation must be opposite in sign. Thus, if we agree that $(-3) \cdot (2) = -6$, then it must follow that $(-3) \cdot (-2) = 6$.

Students often forget to distribute the subtraction to terms other than the first one. For example, students will write $(4x + 3) - (2x + 1) = 4x + 3 - 2x + 1 = 2x + 4$ rather than $4x + 3 - 2x - 1 = 2x + 2$.

Students will change the degree of the variable when adding/subtracting like terms. For example, $2x + 3x = 5x^2$ rather than $5x$.

Students may not distribute the multiplication of polynomials correctly and only multiply like terms. For example, they will write $(x + 3)(x - 2) = x^2 - 6$ rather than $x^2 - 2x + 3x - 6$.

● **A.REI.D.11**

Students may believe that the graph of a function is simply a line or curve “connecting the dots,” without recognizing that the graph represents all solutions to the equation. Students may also believe that graphing linear and other functions is an isolated skill, not realizing that multiple graphs can be drawn to solve equations involving those functions.

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Additionally, students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities.

- **A.CED.A.1**

Students may believe that equations of linear, quadratic and other functions are abstract and exist only “in a math book,” without seeing the usefulness of these functions as modeling real-world phenomena.

Additionally, they believe that the labels and scales on a graph are not important and can be assumed by a reader, and that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.

Students may interchange slope and y -intercept when creating equations. For example, a taxi cab costs \$4 for a dropped flag and charges \$2 per mile. Students may fail to see that \$2 is a rate of change and is slope while the \$4 is the starting cost and incorrectly write the equation as $y = 4x + 2$ instead of $y = 2x + 4$.

Given a graph of a line, students use the x -intercept for b instead of the y -intercept.

Given a graph, students incorrectly compute slope as run over rise rather than rise over run. For example, they will compute slope with the change in x over the change in y .

Students do not know when to include the “or equal to” bar when translating the graph of an inequality.

Students do not correctly identify whether a situation should be represented by a linear, quadratic, or exponential function.

Students often do not understand what the variables represent. For example, if the height h in feet of a piece of lava t seconds after it is ejected from a volcano is given by $h(t) = -16t^2 + 64t + 936$ and the student is asked to find the time it takes for the piece of lava to hit the ground, the student will have difficulties understanding that $h = 0$ at the ground and that they need to solve for t .

- **F.IF.B.4***

- **F.IF.B.5***

- **F.IF.B.6**

Students may believe that it is reasonable to input any x -value into a function, so they will need to examine multiple situations in which there are various limitations to the domains.

Students may also believe that the slope of a linear function is merely a number used to sketch the graph of the line. In reality, slopes have real-world meaning, and the idea of a rate of change is fundamental to understanding major concepts from geometry to calculus.

- **F.IF.C.7***

- **F.IF.C.8***

- **F.IF.C.9***

Students may believe that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs.

Students may also believe that skills such as factoring a trinomial or completing the square are isolated within a unit on polynomials, and that they will come to understand the usefulness of these skills in the context of examining characteristics of functions.

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Additionally, student may believe that the process of rewriting equations into various forms is simply an algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited.

- **F.BF.A.1**

Students may believe that the best (or only) way to generalize a table of data is by using a recursive formula.

Students naturally tend to look “down” a table to find the pattern but need to realize that finding the 100th term requires knowing the 99th term unless an explicit formula is developed.

Students may also believe that arithmetic and geometric sequences are the same. Students need experiences with both types of sequences to be able to recognize the difference and more readily develop formulas to describe them.

- **F.BF.B.3**

Students may believe that the graph of $y = (x - 4)^3$ is the graph of $y = x^3$ shifted 4 units to the left (due to the subtraction symbol). Examples should be explored by hand and on a graphing calculator to overcome this misconception.

Students often confuse the shift of a function with the stretch of a function.

Students may also believe that *even* and *odd* functions refer to the exponent of the variable, rather than the sketch of the graph and the behavior of the function.

Additionally, students may believe that all functions have inverses and need to see counter examples, as well as examples in which a non-invertible function can be made into an invertible function by restricting the domain.

For example, $f(x) = x^2$ has an inverse $(f^{-1}(x) = \sqrt{x}$ provided that the domain is restricted to $x \geq 0$.

- **F.LE.A.3**

Students may believe that all functions have a first common difference and need to explore to realize that, for example, a quadratic function will have equal second common differences in a table.

Students may also believe that the end behavior of all functions depends on the situation and not the fact that exponential function values will eventually get larger than those of any other polynomial functions.

- **N.RN.B.3**

Some students may believe that both terminating and repeating decimals are rational numbers, without considering non-repeating and non-terminating decimals as irrational numbers.

Students may also confuse irrational numbers and complex numbers, and therefore mix their properties. In this case, students should encounter examples that support or contradict properties and relationships between number sets (i.e., irrational numbers are real numbers and complex numbers are non-real numbers. The set of real numbers is a subset of the set of complex numbers).

By using false extensions of properties of rational numbers, some students may assume that the sum of any two irrational numbers is also irrational. This statement is not always true (e.g., $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$, a rational number), and therefore, cannot be considered as a property.

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Fluency Recommendations:

Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.A.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.A.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

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District/School Tasks		District/School Primary and Supplementary Resources and Technology Integration	
<p>PARCC Released Items http://www.parc-assessment.org/released-items</p> <p>NJDOE Digital Item Library https://nj.digitalitemlibrary.com/home</p> <p>NJSLA Mathematics Evidence Statements https://docs.google.com/spreadsheets/d/18M5r1jk4P729fTpAIWAzrw1gE6tken233I-Yk0U712M/edit#gid=554025491</p> <p>LinkIt! Form A, B, & C</p>		<p>Textbook HS Flip Book: http://community.ksde.org/Default.aspx?tabid=5646</p> <p>IXL https://www.ixl.com/</p> <p>Khan Academy https://www.khanacademy.org/</p> <p>North Carolina Dept of Ed. Wikispaces: http://maccss.ncdpi.wikispaces.net/High+School</p> <p>PARCC Resources: http://www.parc-assessment.org/assessments/test-design/mathematics/math-test-specifications-documents</p> <p>101 Math Discourse Questions: http://www.casamples.com/downloads/100MathDiscourseQuestions_Printable.pdf</p> <p>Asking Effective Questions http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/CBS_AskingEffectiveQuestions.pdf</p> <p>Diversity, Equity & Inclusion Educational Resources https://www.nj.gov/education/standards/dei/</p>	
Instructional Best Practices and Exemplars			
<ol style="list-style-type: none"> 1. Identifying similarities and differences 2. Summarizing and note taking 3. Reinforcing effort and providing recognition 4. Homework and practice 5. Nonlinguistic representations 		<ol style="list-style-type: none"> 6. Cooperative learning 7. Setting objectives and providing feedback 8. Generating and testing hypotheses 9. Cues, questions, and advance organizers 10. Manage response rates 	
Vocabulary			
Completing the square Domain Exponential function Extreme values Factoring Function Intercepts	Interval Irrational number Linear function Maxima Maximum Minima	Minimum Parameter Polynomial Range Rational number Relative maximum	Relative minimum Root Quantitative relationship Quadratic function Symmetry Zeros

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9.1 Personal Financial Literacy, 9.2 Career Awareness, Exploration, Preparation and Training & 9.4 Life Literacies and Key Skills

- 9.1.12.CDM.2: Compare and contrast the advantages and disadvantages of various types of mortgages.
9.1.12.CDM.4: Identify issues associated with student loan debt, requirements for repayment, and consequences of failure to repay student loan debt.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
9.4.12.TL.3: Analyze the effectiveness of the process and quality of collaborative environments.

The implementation of the 21st Century skills and standards for students of the Winslow Township District is infused in an interdisciplinary format in a variety of curriculum areas that include, English language Arts, Mathematics, School Guidance, Social Studies, Technology, Visual and Performing Arts, Science, Physical Education and Health, and World Language.

Additional opportunities to address 9.1, 9.2 & 9.4:

Philadelphia Mint

<https://www.usmint.gov/learn/kids/resources/educational-standards>

Different ways to teach Financial Literacy.

<https://www.makeuseof.com/tag/10-interactive-financial-websites-teach-kids-money-management-skills/>

Suggested Modifications for Special Education/504

Students with special needs: The students' needs will be addressed on an individual and grade level using a variety of modalities. Accommodations will be made for those students who need extra time to complete assignments. Support staff will be available to aid students related to IEP specifications. 504 accommodations will also be attended to by all instructional leaders. Physical expectations and modifications, alternative assessments, and scaffolding strategies will be used to support this learning. The use of Universal Design for Learning (UDL) will be considered for all students as teaching strategies are considered.

- | | |
|--|---|
| <input type="checkbox"/> Provide the opportunity to re-take tests | <input type="checkbox"/> Individual Intervention/Remediation |
| <input type="checkbox"/> Modify activities/assignments/projects/assessments | <input type="checkbox"/> Additional Support Materials |
| <input type="checkbox"/> Breakdown activities/assignments/projects/assessments into manageable units | <input type="checkbox"/> Guided Notes |
| <input type="checkbox"/> Additional time to complete activities/assignments/projects/assessments | <input type="checkbox"/> Graphic Organizers |
| <input type="checkbox"/> Provide an option for alternative activities/assignments/projects/assessments | <input type="checkbox"/> Adjust Pacing of Content |
| <input type="checkbox"/> Modify Content | <input type="checkbox"/> Increase one on one time |
| <input type="checkbox"/> Modify Amount | <input type="checkbox"/> Peer Support |
| <input type="checkbox"/> Small Group Intervention/Remediation | <input type="checkbox"/> Other Modifications for Special Education: |

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Suggested Modifications for At-Risk Students	
<p>Formative and summative data will be used to monitor student success. At first signs of failure, student work will be reviewed to determine support. This may include parent consultation, basic skills review and differentiation strategies. With considerations to UDL, time may be a factor in overcoming developmental considerations</p>	
<input type="checkbox"/> Provide the opportunity to re-take tests <input type="checkbox"/> Increase one on one time <input type="checkbox"/> Oral prompts can be given <input type="checkbox"/> Using visual demonstrations, illustrations, and models <input type="checkbox"/> Give directions/instructions verbally and in simple written format <input type="checkbox"/> Peer Support <input type="checkbox"/> Modify activities/assignments/projects/assessments <input type="checkbox"/> Additional time to complete activities/assignments/projects/assessments <input type="checkbox"/> Provide an option for alternative activities/assignments/projects/assessments	<input type="checkbox"/> Modify Content <input type="checkbox"/> Modify Amount <input type="checkbox"/> Adjust Pacing of Content <input type="checkbox"/> Small Group Intervention/Remediation <input type="checkbox"/> Individual Intervention/Remediation <input type="checkbox"/> Additional Support Materials <input type="checkbox"/> Guided Notes <input type="checkbox"/> Graphic Organizers <input type="checkbox"/> Other Modifications for Students At-Risk:
Suggested for English Language Learners	Suggested Modifications for Gifted Students
<p>All WIDA Can Do Descriptors can be found at this link: https://wida.wisc.edu/teach/can-do/descriptors</p> <input type="checkbox"/> Grades 9-12 WIDA Can Do Descriptors: <ul style="list-style-type: none"> <input type="checkbox"/> Listening <input type="checkbox"/> Speaking <input type="checkbox"/> Reading <input type="checkbox"/> Writing <input type="checkbox"/> Oral Language <p>Students will be provided with accommodations and modifications that may include:</p> <ul style="list-style-type: none"> • Relate to and identify commonalities in mathematics studies in student’s home country • Assist with organization • Use of computer • Emphasize/highlight key concepts • Teacher Modeling • Peer Modeling • Label Classroom Materials - Word Walls 	<p>Students excelling in mastery of standards will be challenged with complex, high level challenges related to the topic.</p> <ul style="list-style-type: none"> • Raise levels of intellectual demands • Require higher order thinking, communication, and leadership skills • Differentiate content, process, or product according to student’s readiness, interests, and/or learning styles • Provide higher level texts • Expand use of open-ended, abstract questions • Critical and creative thinking activities that provide an emphasis on research and in-depth study • Enrichment Activities/Project-Based Learning/ Independent Study <p>Additional Strategies may be located at the links:</p> <ul style="list-style-type: none"> ❖ Gifted Programming Standards ❖ Webb’s Depth of Knowledge Levels and/or Revised Bloom’s Taxonomy ❖ REVISED Bloom’s Taxonomy Action Verbs

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Suggested Activities			
<input type="checkbox"/> Do Now/Warm-Up <input type="checkbox"/> Whole Group <input type="checkbox"/> Small Groups	<input type="checkbox"/> Guided Practice <input type="checkbox"/> Independent Practice	<input type="checkbox"/> Centers <input type="checkbox"/> Intervention/Remediation <input type="checkbox"/> Projects	<input type="checkbox"/> Academic Games <input type="checkbox"/> Other Suggested Activities:
Interdisciplinary Connections			
<p>Big Ideas Real-Life STEM Videos and Performance Tasks</p> <p>Interdisciplinary Connections: ELA</p> <p>NJSLSA.R1. Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.</p> <p>NJSLSA.W2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content</p> <p>NJSLSA.L1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking</p> <p>SL.9-10.4: Present information, findings and supporting evidence clearly, concisely and logically. The content, organization, development and style are appropriate to task, purpose and audience.</p> <p>NJSLSA.L6: Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.</p>			
Integration of Computer Science and Design Thinking NJSLS 8			
<p>8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.</p> <p>8.1.12.AP.2: Create generalized computational solutions using collections instead of repeatedly using simple variables.</p> <p>8.1.12.AP.8: Evaluate and refine computational artifacts to make them more usable and accessible.</p> <p>8.1.12.DA.1: Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.</p> <p>8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.</p> <p>8.2.12.ETW.2: Synthesize and analyze data collected to monitor the effects of a technological product or system on the environment.</p> <p>8.2.12.ETW.3: Identify a complex, global environmental or climate change issue, develop a systemic plan of investigation, and propose an innovative sustainable solution.</p>			